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DARWIN MEETS *THE LOGIC OF DECISION*: CORRELATION IN EVOLUTIONARY GAME THEORY*

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The proper treatment of correlation in evolutionary game theory has unexpected connections with recent philosophical discussions of the theory of rational decision. *The Logic of Decision* (Jeffrey 1983) provides the correct framework for correlated evolutionary game theory and a variant of "ratifiability" is the appropriate generalization of "evolutionarily stable strategy". The resulting theory unifies the treatment of correlation due to kin, population viscosity, detection, signaling, reciprocal altruism, and behavior-dependent contexts. It is shown that (1) a strictly dominated strategy may be selected, and (2) under conditions of perfect correlation a strictly efficient strategy *must* be selected.

1. Introduction. The theory of rational deliberation and the theory of evolution both deal with processes which tend to move in the direction of a provisional optimum. In both areas, strategic interaction leads to complex game theoretic situations where the provisional optimum may be a moving target, and where equilibrium considerations must be intro-

duced. In both disciplines, theories initially developed under simplifying independence assumptions need to be extended to deal with correlation. However, these extensions lead in different directions. In this regard, the proper treatment of correlation in evolutionary game dynamics has un-expected connections with philosophical discussions of the correct theory of rational decision.

The evolutionary game theory of J. Maynard Smith is built on the sim-plifying assumption of pairwise encounters between individuals randomly selected from the population. But, as biologists well know, such random pairing is not the norm in the real world. Nonrandom pairings can and do occur in a number of different ways. A truly general version of evo-lutionary game theory should provide a framework within which all non-random pairings can be accommodated. This simple observation leads to striking consequences. Correlation in the evolutionary setting calls for a different theory than correlation in the theory of rational choice.

In *The Logic of Decision* R. Jeffrey (1983) proposed a novel form of decision theory according to which the weights used in calculating ex-pected utility of an act are not the unconditional probabilities of states of the world but rather the probabilities conditional on the act in question. Jeffrey has criticized the results of his theory in certain problematic cases, and has proposed a modification based on a new notion of *ratifiability*. In fact the conditional expected utility of *The Logic of Decision* is a cor-rect model for calculating expected fitness in generalized evolutionary game theory, and a variant of ratifiability is important for defining the appropriate generalization of Maynard Smith's concept of an *evolution-arily stable strategy*.

Results problematic for the theory as a theory of rational choice make perfect sense in the context of population dynamics. Consequently, the relevant equilibrium concept for generalized evolutionary game theory will differ from either the Nash equilibrium of classical game theory or the correlated equilibrium more recently introduced by R. Aumann (1974,1987). The case of one-shot prisoner's dilemma is a striking ex-ample. Under favorable conditions of correlation, the strategy of coop-eration can take over the entire population. The example generalizes in ways that (1) show the gulf between correlated evolutionary game theory and correlated economic game theory; and (2) show how correlation can maximize the average fitness of the population.

2. "The Logic of Decision". In *The Logic of Decision*, Jeffrey intro-duced a new framework for decision theory, which was meant to modify and generalize the classic treatment of Savage (1954). Savage sharply distinguishes acts, states of the world and consequences. All utility re-sides in consequences. Acts together with states jointly determine con-

sequences, so acts can be taken to be functions from states to conse-
quences. All uncertainty about the consequences of one's acts is carried
by the states of the world. States are thus the points in the probability
space associated with the model and acts can be conceived of as random
variables mapping states to the utilities of the associated consequences.
The expected utility of an act is then just the expectation of the act so
considered as a random variable. In the special case of only a finite num-
ber of states, we can write the expected utility of an act as a probability
weighted average of the utilities of application of that act to each state:

SAVAGE: Utility(Act) = Σ_i Probability ($State_i$) Utility ($Act(State_i)$).

Jeffrey allowed for the possibility that the act chosen might influence
the probability of the states. He makes no formal distinction between acts,
states and consequences but relies on a Boolean algebra, whose elements
are to be thought of as propositions. Each proposition has a probability
and each proposition with positive probability has a utility. In the appli-
cation of the theory the decision maker can identify a partition of prop-
ositions that represent the alternative possible acts of her decision prob-
lem, and a partition representing alternative states of the world. Jeffrey
takes the expected utility of an act to be a weighted average of the utilities
of act-state conjunctions, with the weighting of the average being the
conditional probability of state conditional on act instead of on the un-
conditional probability used in Savage:

JEFFREY: Utility(Act)
$$= \Sigma_i \text{ Probability } [State_i \mid Act] \text{ Utility } [Act \,\&\, State_i].$$

The Jeffrey expected utility makes sense for any element A of the prob-
ability space relative to any finite partition $\{S_i\}$ whether or not the former
is intuitively an act and the latter a partition of states. Furthermore, for
fixed A, the expected utility of A comes out the same when calculated
relative to any finite partition, thus enabling Jeffrey to dispense with a
formal distinction between acts and states, and to endow all (non-null)
elements of the basic probability algebra with an expected utility as well
as a probability. The expected utility of the whole space is of special
interest. This is the expected utility of the status quo. When the agent is
undecided about which act to do, it can be gotten by expecting over the
expected utilities of the acts in an act partition, so even a state of inde-
cision is assigned an expected utility:

JEFFREY UTILITY of the STATUS QUO: USQ
$$= \Sigma_j \text{ Probability } (Act_j) \text{ Utility}(Act_j).$$

However, difficulties arise when Jeffrey's system is interpreted as a
system for rational decision. The probabilities in question are just the

TABLE 2.1.   MAX'S PAYOFFS

	Moritz Cooperates	Moritz Defects
Max Cooperates	0.9	0
Max Defects	1	0.6

agent's degrees of belief. But then probabilistic dependence between act and state may arise for reasons other than the one that Jeffrey had in mind—that the agent takes the act as tending to bring about the state. The dependence in degrees of belief might rather reflect that an act is evidence for a state obtaining, for instance, because the act and state are symptoms of a common cause. This raises the prospect of basing decisions on spurious correlation. (See Gibbard and Harper 1981; Lewis 1981; Nozick 1970; Skyrms 1980, 1984; and Stalnaker 1981.) Prisoner's dilemma with a clone—or a near clone—is a well-known kind of illustration of the difficulty (Lewis 1979, Gibbard and Harper 1981).

Max and Moritz are apprehended by the authorities and are forced to play the prisoner's dilemma. (For biographical data see Busch 1865.) Each is given the choice to remain silent (=cooperate) or turn state's evidence (=defect). We discuss the decision problem from the point of view of Max, but Moritz's situation is taken to be symmetrical. Max's payoffs depend both on what he does and what Moritz does, and he takes his utilities to be as given in table 2.1. Max also believes that he and Moritz are much alike and although he is not sure what he will do, he thinks he and Moritz are likely to decide the same. His beliefs do not make his act probabilistically independent of that of Moritz even though we assume that they are sequestered so that one act cannot influence the other. We have evidential relevance with causal independence. For definiteness, we assume that Max has the probabilities for joint outcomes given in table 2.2. (Thus, for example, Max's probability that he and Moritz both cooperate is 0.45, and his conditional probability of Moritz's cooperation given that he does is 0.9.)

If Max applies Savage's theory and takes Moritz's acts as constituting his own states, he will take the states as equiprobable and calculate Savage expected utility of his cooperating as $(0.5)(0.9) = 0.45$ and Savage expected utility of his defecting as $(0.5)(1) + (0.5)(0.6) = 0.8$. He will maximize Savage expected utility by defecting. This conclusion is independent of the probabilities assumed since defection *strictly dominates*

TABLE 2.2.   MAX'S JOINT PROBABILITIES

	Moritz Cooperates	Moritz Defects
Max Cooperates	0.45	0.05
Max Defects	0.05	0.45

cooperation in the pay-off matrix—that is, whatever Moritz does, Max is better off defecting.

But if Max applies Jeffrey expected utility, using conditional probabilities as weights, he will calculate the Jeffrey expected utility of cooperating as $(0.9)(0.9) = 0.81$ and the Jeffrey expected utility of defecting as $(0.1)(1) + (0.9)(0.6) = 0.64$. He will maximize Jeffrey expected utility by cooperating. Maximization of Jeffrey expected utility selects a strictly dominated act.

In response to these difficulties, Jeffrey introduced a new concept in the second edition of *The Logic of Decision*: that of ratifiability. (For related ideas, see Eells 1982, 1984.) Jeffrey's idea was that during the process of deliberation, the probabilities conditional on the acts might not stay constant, but instead evolve in such a way that the spurious correlation was washed out. In other words, it is assumed that at the end of deliberation the states will be probabilistically independent of the acts. If so, the Jeffrey expected utility will be equal to the Savage expected utility. Thus, in the previous example expected utility at the end of deliberation would respect dominance and defection would then maximize Jeffrey expected utility.

Consider the probability measure that an agent would have on the brink of doing act A, and let U_A be the Jeffrey expected utility calculated according to this probability. An act A is said to be *ratifiable* just in case:

$$U_A(A) \geq U_A(B) \text{ for all } B \text{ different from } A.$$

We will say that it is *strictly ratifiable* if the inequality is strict:

$$U_A(A) > U_A(B) \text{ for all } B \text{ different from } A.$$

Jeffrey suggested that a choiceworthy act should be a ratifiable one. (For further discussion see Skyrms 1990a,b.)

The reason for talking about "the brink" is that when the probability of an act is equal to one, the probabilities conditional on the alternative acts have no natural definition. The idea of ratifiability, so expressed, is ambiguous according to how "the brink" is construed. In section 4 we specify ratifiability as an inequality holding throughout some neighborhood of the point at which the probability of the act is equal to one.

3. Replicator Dynamics and Evolutionarily Stable Strategies. A standard model is widely used to model the dynamics of evolutionary games. Individuals can have various alternative "strategies" or dispositions to act in certain ways in pairwise encounters. These strategies are genetically determined. Reproduction is asexual and individuals breed true. Each individual engages in one contest per generation, and plays its strategy. Payoffs are in terms of evolutionary fitness (expected number of off-

spring). The payoff for an individual playing strategy A_i against one playing strategy A_j is written as $U(A_i|A_j)$. The population is very large (effectively infinite). Individuals are paired at random.

Let us write $p(A_i)$ for the proportion of the population playing strategy A_i. This is also the probability that an individual playing A_i is selected in a random selection from the population. Then, under the foregoing assumptions, the expected fitness for an individual playing A_i is determined by averaging over all the strategies that A_i may be played against: $U(A_i)$ $= \Sigma_j p(A_j) \, U(A_i|A_j)$. The average fitness of the population U is calculated by averaging over all strategies: $U = \Sigma_i p(A_i) \, U(A_i)$. If the population is large enough, then the expected number of offspring to individuals playing strategy A_i, $U(A_i)$, is with high probability close to the actual number of offspring. The population is assumed to be large enough that a useful approximation can be derived by studying the deterministic map which identifies the expected number of offspring to individuals playing a strategy with the actual number of offspring. (For a careful discussion of this reasoning see Boylan 1992.) Under this assumption the proportion of the population playing a strategy in the next generation p' is equal to:

$$p'(A_i) = p(A_i) \, [U(A_i)/U].$$

Considered as a dynamical system with discrete time, the population evolves according to the difference equation:

$$p'(A_i) - p(A_i) = p(A_i) \, [U(A_i) - U]/U.$$

If the time between generations is small this may be approximated by a continuous dynamical system governed by the differential equation:

$$dp(A_i)/dt = p(A_i) \, [U(A_i) - U]/U.$$

Provided average fitness of the population is positive, the orbits of this differential equation on the simplex of population proportions for various strategies are the same as those of the simpler differential equation:

$$dp(A_i)/dt = p(A_i) \, [U(A_i) - U]$$

although the velocity along the orbits may differ (see van Damme 1987, sec. 9.4).[1] This latter equation was introduced by Taylor and Jonker (1978). It was later studied by Zeeman (1980), Bomze (1986), Hofbauer and Sigmund (1988), and Nachbar (1990). Schuster and Sigmund (1983) find it at various levels of biological dynamics and call it the *replicator dynamics*.

[1]The equivalence would fail if we considered evolutionary games played between two different populations because of differences in the average fitnesses of the two populations. The "Battle of the Sexes" game provides an example. See Maynard Smith (1982, appendix J) and Hofbauer and Sigmund (1988, part 7).

A *dynamic equilibrium* is a fixed point of the dynamics under consideration. In the case of discrete time, it is a point **x** of the state space that the dynamics maps onto itself. For continuous time, it is a state $\mathbf{x} = \langle x_1, \ldots, x_i, \ldots \rangle$ such that $dx_i/dt = 0$, for all i. An equilibrium **x** is *stable* if points near to it remain near. More precisely, **x** is stable if for every neighborhood V of **x**, there is a neighborhood V' of **x** such that if the state **y** is in V' at time $t = 0$, it remains in V for all time $t > 0$. An equilibrium **x** is *strongly stable* (or asymptotically stable) if nearby points tend toward it. That is, to the definition of stability we add the clause that the limit as t goes to infinity of $\mathbf{y}(t) = \mathbf{x}$. The states of interest are vectors of population proportions. We treat these formally as probabilities. Since these must add to one, the state space is a probability simplex. We will say that an equilibrium is *globally stable* in the replicator dynamics if it is the dynamical limit as time goes to infinity of every point in the interior of the state space.

Taylor and Jonker introduced the replicator dynamics to provide a dynamical foundation for the equilibrium notion—*evolutionarily stable strategy*—introduced informally in Maynard Smith and Price (1973) and formally in Maynard Smith and Parker (1976). Informally, if all members of the population adopt an evolutionarily stable strategy then no mutant can invade. Maynard Smith and Parker propose the following formal realization of this idea:

> Strategy x is *evolutionarily stable* just in case $U(x|x) > U(y|x)$ or $U(x|x) = U(y|x)$ and $U(x|y) > U(y|y)$ for all y different from x. Equivalently, x is evolutionarily stable if
>
> 1. $U(x|x) \geq U(y|x)$; and
> 2. if $U(x|x) = U(y|x)$, then $U(x|y) > U(y|y)$.

Maynard Smith and Price's set of strategies included all randomized strategies that can be made from members of the set. In the Taylor-Jonker framework individuals play pure (nonrandom) strategies and the place of randomized strategies is taken by mixed or polymorphic states of the population where different proportions of the population play different pure strategies. The mathematics remains the same, but the interpretation changes: Mixed states of the population satisfy the definition as evolutionarily stable states. In this case, the notion of an evolutionarily stable state is stronger than that of a strongly stable equilibrium point in the replicator dynamics. Taylor and Jonker show that every evolutionarily stable state is a strongly stable equilibrium point in the replicator dynamics but give an example where a mixed state is a strongly stable equilibrium point but not an evolutionarily stable state.

This dynamics exhibits close connections with individual rational de-

cision theory and with the theory of games. Following is a sketch of the most important parts of the picture. First, notice that if we think of evolutionary fitness as utility and population proportion as probability, then the formula for expected fitness of an individual playing a strategy is the same as that for the *Savage expected utility* of an act.

In a two-person, finite, noncooperative, normal form game there are a finite number of players and each player has a finite number of possible strategies. Each possible combination of strategies determines the payoffs for each of the players. (The games are to be thought of as noncooperative. No communication or precommitment occurs before the players make their choices.) A specification of the number of strategies for each of the two players and the pay-off function determines the game. A *Nash equilibrium* of the game is a strategy combination such that no player does better on any unilateral deviation. We extend players' possible acts to include randomized choices at specified probabilities over the originally available acts. The new randomized acts are called *mixed strategies*, and the original acts are called *pure strategies*. The payoffs for mixed strategies are defined as their expected values using the probabilities in the mixed acts to define the expectation. We will assume that mixed acts are always available. Then every finite, noncooperative, normal form game has a Nash equilibrium.

For any evolutionary game given by a fitness matrix, there is a corresponding symmetric two-person noncooperative game. (It is symmetric because the payoff for one strategy played against another is the same if row plays the first and column plays the second, or conversely. The identity of the players does not matter.) If $\mathbf{x}$ is an evolutionarily stable state, then $\langle \mathbf{x}, \mathbf{x} \rangle$ is—by condition 1 above—a symmetric Nash equilibrium of that two-person noncooperative game. Condition 2 adds a stability requirement. If $\langle \mathbf{x}, \mathbf{x} \rangle$ is a Nash equilibrium of the two-person game, then $\mathbf{x}$ is a dynamic equilibrium of the replicator dynamics, but not conversely. If $\mathbf{x}$ is a stable dynamic equilibrium of the replicator dynamics, then $\langle \mathbf{x}, \mathbf{x} \rangle$ is a Nash equilibrium of the two-person game, but not conversely. (For details and proofs see van Damme 1987.)

The foregoing model motivating the replicator dynamics relies on many simplifying assumptions and idealizations which might profitably be questioned. Here, however, we focus on the assumption of random pairing. There is no mechanism for random pairing in nature and ample reason to believe that pairing is often not random (Hamilton 1964). Random pairing provides a certain mathematical simplicity and striking connections with the Nash equilibrium concept of the von Neumann and Morgenstern (1947) theory of games, but a theory which accommodates all kinds of nonrandom pairing would be more adequate for realistic models. How should we formulate such a general theory?

4. Darwin Meets *The Logic of Decision*. Let us retain the model of the previous section with the single modification that pairing is not random. Nonrandom pairing might occur because individuals using the same strategies tend to live together, or because individuals using different strategies present some sensory cue that affects pairing, or for other reasons. We want a framework general enough to accommodate all kinds of non-random pairing.

The characterization of a state of the biological system, then, must specify conditional proportions $p(A_j|A_i)$ consistent with the population proportions—which give the proportion of individuals using strategy A_i which will interact with individuals using strategy A_j. Now the expected fitness for an individual playing A_i is derived by averaging over all the strategies that A_i may be played against, using the conditional proportions rather than the unconditional proportions as weights of the average: $U(A_i)$ $= \Sigma_j p(A_j|A_i) \ U(A_i|A_j)$. Formally, this is Jeffrey's move from Savage to Jeffrey expected utility.

The average fitness of the population is determined by averaging over the strategies using the proportions of the population playing them as weights: $U = \Sigma_i p(A_i) \ U(A_i)$. This is the Jeffrey expected utility of the status quo. *The replicator dynamics then goes exactly as before with the proviso that utility be read as Jeffrey expected utility calculated according to the conditional pairing proportions.*

The dynamics are complicated if the conditional pairing proportions are subject to dynamical evolution. This will often be the case in realistic models, and in certain cases may be forced upon us by the requirement that the pairing proportions be consistent with the population proportions. To take an extreme case, suppose that two strategies are initially represented in equal proportions in the population and suppose each strategy strongly tends to pair with the other. If the fitnesses are such that strategy one flourishes and strategy two is driven toward extinction, the strong anticorrelation cannot be maintained because not enough strategy-two individuals are available to pair with all the strategy-one players at a given time. No such consistency problems occur in maintaining strong positive correlations between strategies in two-strategy games. In the case just described each strategy could almost always be paired with itself. However, the specific biological motivation for correlation could easily motivate a dynamical evolution of the conditional pairing proportions in this case as well.

What are the relevant notions of equilibrium and stable equilibrium for pure strategies in correlated evolutionary game theory? Every pure strategy is a dynamical equilibrium in the replicator dynamics because its potential competitors have zero population proportion. Maynard Smith introduced the nontrivial equilibrium concept of *evolutionarily stable*

strategy into evolutionary game theory. A strategy is evolutionarily stable if, for all potential competitors, either it does better against itself than the competitor or it does better against the competitor than the competitor does against itself. But the notion is relevant only in the context of the random pairing assumption. It does not take correlation into account. Where we have correlation, being an evolutionarily stable strategy in Maynard Smith's sense is neither necessary nor sufficient for being a dynamically stable equilibrium (as will be shown by example). We want a stability concept that gives correlation due weight, and that applies in the general case when the conditional pairing proportions are not fixed during the dynamical evolution of the population. For such a notion we return to Jeffrey's concept of ratifiability.

Transposing Jeffrey's idea to this context, a pure strategy is ratifiable if it maximizes expected fitness when it is on the brink of fixation. (The population is at a state of fixation of strategy A when 100 percent of the population uses strategy A.) This means that in some neighborhood of the state of fixation of the strategy, the strategy maximizes expected utility in that state (where the state of the system is specified in the model so as to determine both the population proportions and the conditional pairing proportions).

Let us focus on models where the conditional pairing proportions are functions of the population proportions, so that the population proportions specify the state of the system and the replicator dynamics specifies a complete dynamics for the system. Since we are interested in strong stability, the natural concept to consider is that of strict ratifiability. Let $\mathbf{x}$ be vector of population proportions specifying the state of the system; let $\mathbf{a}$ be the state of the system which gives pure strategy A probability one; let $U_\mathbf{x}(B)$ be the expected fitness of B when the system is in state $\mathbf{x}$, and $U_\mathbf{x}$ be the average fitness of the population in state $\mathbf{x}$. Then a pure strategy A is strictly ratifiable if for all pure strategies B different from A:

$$U_\mathbf{x}(A) > U_\mathbf{x}(B)$$

for all $\mathbf{x} \neq \mathbf{a}$ in some neighborhood of $\mathbf{a}$ (the point of fixation of A).

There is, however, reason to explore a weaker variation on the general theme of ratifiability. Here we ask only that the expected fitness of A is higher than that of the average fitness of the population throughout a neighborhood of the point of fixation of A. What is required to hold throughout the neighborhood is not that A is optimal but only that A is adaptive, that A is better than the status quo. I will call this concept adaptive ratifiability. A pure strategy A is adaptive-ratifiable if:

$$U_\mathbf{x}(A) > U_\mathbf{x}$$

for all $\mathbf{x} \neq \mathbf{a}$ in some neighborhood of $\mathbf{a}$ (the point of fixation of A).

TABLE 4.1.

	Strategy 1	Strategy 2	Strategy 3
Strategy 1	3	3	3
Strategy 2	3	0	4
Strategy 3	3	4	0

Obviously strict ratifiability entails adaptive ratifiability since the average population fitness U_x is an average of the fitnesses of the pure strategies $U_x(B_i)$. For an example that shows that adaptive ratifiability does not entail strict ratifiability, consider the fitness matrix in table 4.1 together with the assumption of random pairing. Then strategy 1 is not strictly ratifiable, because wherever $p(S2)/p(S3) > 3$, strategy 3 has higher fitness than strategy 1, and wherever $p(S3)/p(S2) > 3$, strategy 2 has greater fitness than strategy 1. Strategies 2 and 3 each prosper when rare relative to the other, but the rare strategy cannot make so great an impact on the average fitness of the population as the other strategy which cannot prosper. In fact, the average fitness of the population is at its unique maximum at the point of fixation of strategy 1. Strategy 1 is therefore adaptive-ratifiable.

We can extend the concept of adaptive ratifiability from pure strategies to mixed states of the population. If **p** is the vector of population proportions, then $U_{\mathbf{p}} = \Sigma_i p(A_i) U(A_i)$ is the average fitness of the population in mixed state **p**. Let **x** be another vector of population proportions, and consider $U_x(\mathbf{p}) = \Sigma_i p(A_i) U_x(A_i)$. This quantity is what the average population fitness would be if the expected fitness of each pure strategy A were determined by vector **x** but the average fitness were determined by vector **p**. (Alternatively, it could be thought of as the payoff to a mutant playing a true mixed strategy **p** in a population in state **x**.)

We can then say that **p** *is an adaptively ratifiable state if*:

$$U_x(\mathbf{p}) > U_x,$$

for all $\mathbf{x} \neq \mathbf{p}$ in some neighborhood of **p**.

Two facts already known from the analysis of conventional evolutionary game theory show that adaptive ratifiability plays a central role in correlated evolutionary game theory. The first is that adaptive ratifiability generalizes the evolutionarily stable strategies of Maynard Smith and Price:

> In evolutionary game theory with random pairing, a state is Evolutionarily Stable if and only if it is Adaptive-Ratifiable. (Van Damme 1987, theorem 9.2.8)

The second is that adaptive ratifiability guarantees strong stability in the replicator dynamics:

> If a pure strategy is Adaptive-Ratifiable, then it is an attracting equi-
> librium in the replicator dynamics. (Ibid., theorem 9.4.8)

On this basis we take adaptive ratifiability to be the natural generalization
of evolutionarily stable state in correlated evolutionary game theory. Thus
we see that three characteristic features of Jeffrey's discussion of rational
decision—Jeffrey expected utility, expected utility of the status quo, and
ratifiability—play important parts in correlated evolutionary game theory.

5. Simple Examples. *5.1. Example 1.* Suppose that the fitnesses for
pairwise encounters are given by the pay-off matrix for the prisoner's
dilemma game played by Max and Moritz. (These are one-shot prisoner's
dilemma games—not the indefinitely repeated prisoner's dilemma games
widely discussed in the literature—and defection is the unique evolu-
tionary stable strategy as defined by Maynard Smith.) Starting from any
mixed population, the replicator dynamics with random pairing converges
to a population of 100 percent defectors. Now consider the extreme case
of prisoner's dilemma with a clone; individuals are paired with like-minded
individuals with perfect correlation. The conditional proportions are $p(C|C)$
$= p(D|D) = 1$ and $p(C|D) = p(D|C) = 0$, remaining fixed at these values
during the evolution of the system. With perfect correlation the expected
fitness for a cooperator is 0.9 and that of a defector is 0.6. The pure
strategy of cooperation is strictly ratifiable and therefore adaptive-ratifiable.
It is a strongly stable equilibrium in the replicator dynamics, and that
dynamics carries any initial population with some positive proportion of
cooperators to a population with 100 percent cooperators. Under these
conditions of correlation, Maynard Smith's definition of evolutionarily
stable strategy is no longer appropriate. Although defection is an evo-
lutionarily stable strategy and cooperation is not, cooperation is a dynam-
ically globally stable equilibrium—that is, its basin of attraction includes
all of the interior of the state space. This example shows in the simplest
way how difficulties for Jeffrey expected utility in the theory of rational
choice become strengths in the context of correlated evolutionary game
theory.

   5.2. Example 2. Correlation is usually not perfect and the relevant
conditional probabilities may depend on population proportions. The spe-
cifics depend on how correlation is supposed to arise. Correlation may
be established by some sort of sensory detection. For instance, cooper-
ators and defectors might emit different chemical markers. Suppose cor-
relation arises as follows. At each moment there is a two-stage process.
First, individuals are randomly paired from the population. If a cooperator
detects another cooperator, they interact. If not, no interaction occurs,

for we assume here that defectors wish to avoid each other as much as cooperators wish to avoid them. Then the members of the population that did not pair on the first try randomly pair among themselves; they give up on detection and interact with whomever they are paired. We assume here that detection accuracy is perfect, so that imperfect correlation among cooperators is due entirely to the possibility of initial failure to meet with a like-minded individual. (This assumption would obviously be relaxed in a more realistic model, as would the assumption that individuals would simply give up on detection after just one try.) The conditional probabilities that arise from this two-stage process then depend on population frequencies as follows:

$$p(C|C) = p(C) + [\{1 - p(C)\} \{p(C) - p(C)^2\}]/[1 - p(C)^2];$$

$$p(D|D) = [1 - p(C)]/[1 - p(C)^2].$$

Using the payoffs for prisoner's dilemma of section 2, the expected fitnesses (=Jeffrey utilities) are:

$$U(C) = 0.9 [p(C) + [\{1 - p(C)\} \{p(C) - p(C)^2\}]/[1 - p(C)^2]];$$

$$U(D) = 1 - [\{0.4 (1 - p(C)\}/\{1 - p(C)^2].$$

Figure 5.1 presents the expected fitnesses of cooperation and defection as a function of the proportion of cooperators in the population. In a population composed of almost all defectors, hardly anyone pairs on the first stage and almost all cooperators end up pairing with defectors as do almost all defectors. The limiting expected fitnesses as defection goes to fixation are just those on the right column of the fitness matrix: $U(D) = 0.6$ and $U(C) = 0$. Defection is strictly ratifiable; a population composed entirely of defectors is strongly stable in the replicator dynamics.

However, defection is not the only strictly ratifiable pure strategy. Cooperation qualifies as well. As the population approaches 100 percent cooperators, cooperators almost always pair with cooperators at the first stage. Defectors random pair with those left at the second stage, but not many cooperators are left. The result is that the expected fitness of cooperation exceeds that of defection. An unstable mixed equilibrium results where the fitness curves cross at $p(C) = 0.75$.

This example illustrates a general technique for obtaining correlated pairing by superimposing a "filter" on a random-pairing model. It also shows that nothing is especially pathological about multiple strictly ratifiable strategies in evolutionary game theory.

5.3. Example 3. For an example of a game with no adaptively ratifiable pure strategies in essentially the same framework, consider the fitness matrix in table 5.1 and the same model of frequency dependent cor-
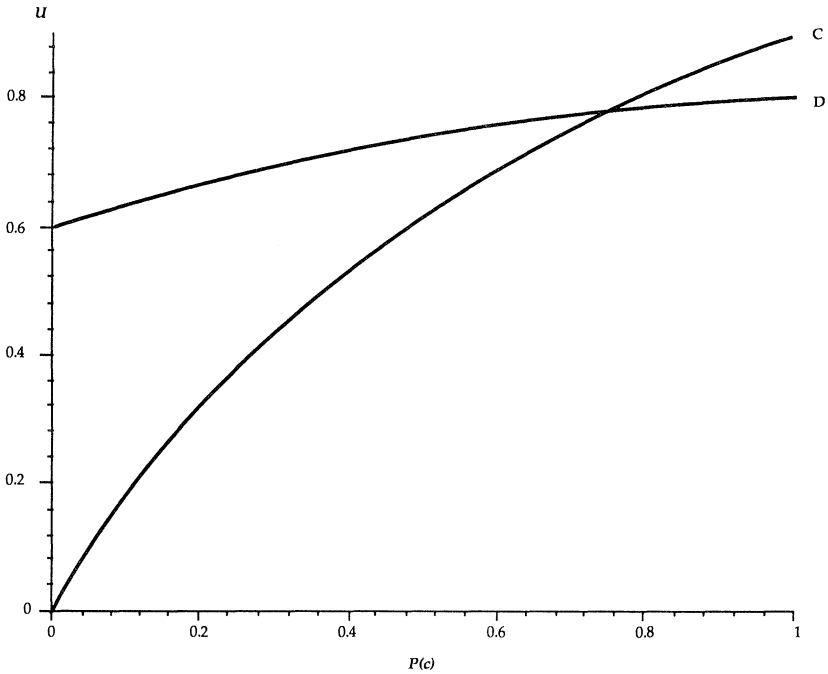
Figure 5.1.

relation except that individuals try to pair with individuals of the other
type at the first stage of the pairing process. Here strategy 1 does better
in a population composed mostly of individuals following strategy 2 and
strategy 2 does better in a population composed of individuals following
mostly strategy 1. The replicator dynamics carries the system to a stable
state where half the population plays strategy 1 and half plays strategy
2. This is the same polymorphism that one would get in the absence of
correlation, but here both strategies derive a greater payoff in the cor-
related polymorphic equilibrium [$U(S1) = U(S2) = 3/4$] than in the un-
correlated one [$U(S1) = U(S2) = 1/2$].

5.4. Example 4. This example departs from the preceding framework.

TABLE 5.1.   FITNESS

	Strategy 1	Strategy 2
Strategy 1	0	1
Strategy 2	1	0

The population is finite, the dynamics are discrete and the population proportions are not sufficient to specify the state of the system. As Hamilton (1964) emphasizes, correlated interactions may take place in the absence of detection or signals when like individuals cluster together spatially. Hamilton discusses nondispersive or "viscous" populations where individuals living together are more likely to be related. In replicator models, relatedness is an all or nothing affair and the effects of viscosity can be striking.

For the simplest possible spatial example, consider a one-dimensional space. A large fixed finite number of individuals are arranged in a row. Each, except those on the ends, has two neighbors. Suppose that in each time period each individual plays a prisoner's dilemma with each of its neighbors and receives the average of the payoffs of these games. We assume that similar individuals cluster, so a group expands or contracts around the periphery. The population proportions will be governed by the discrete replicator dynamics rounded off, and the expansion or contraction of a connected group of like individuals will be determined by the fitnesses of members of that group. The state of the system here depends not only on the population frequency but also on the spatial configuration of individuals playing various strategies.

A single cooperator introduced into space otherwise populated by defectors interacts only with defectors and is eliminated. Scattered isolated cooperators or groups of two are also eliminated. Defection is strongly stable in a sense appropriate for this discrete system. However, if a colony of four contiguous cooperators is introduced in the middle of the space (or three at an end of the space), cooperators will have a higher average fitness than defectors and will increase. Cooperation, however, will not be fixed. The hypothetical last defector interacts only with cooperators and so has a fitness higher than their average fitness. Defectors cannot be completely eliminated. They will persist as predators on the periphery of the community of cooperators. Cooperation fails to be stable. Even though defection is the unique stable pure strategy in this example, many possible initial states of the system are carried to states that include both cooperators and defectors.

These simple models indicate the importance of correlation in evolutionary settings and the striking differences in outcomes of which it can produce. A variety of other models incorporating correlation in one way or another, and fitting within the framework of correlated evolutionary game theory, can be found in the biological, economic and philosophical literature. Some pointers to this literature are given in section 8.

6. Correlation in Evolutionary and in Economic Game Theory. In the absence of correlation, the Nash equilibrium of the rational players

of classical economic game theory and the equilibria of the unconscious adaptive processes of evolutionary game theory almost coincide. To every evolutionary game corresponds a two-player nonzero sum von Neumann-Morgenstern game. We cannot say that **p** is an equilibrium of the replicator dynamics for the evolutionary game iff ⟨**p,p**⟩ is a Nash equilibrium of the von Neumann-Morgenstern game because, as already mentioned, any unmixed population (pure strategy) is an equilibrium of the replicator dynamics.[2] But we can say that if ⟨**p,p**⟩ is a Nash equilibrium of the corresponding two-person game, then **p** is an equilibrium of the replicator dynamics. And if **p** is a stable equilibrium of the replicator dynamics, then ⟨**p,p**⟩ is a Nash equilibrium of the two-person game. For more information on the relation of refinements of the equilibrium concepts in the two settings, see Bomze (1986), Friedman (1991), Nachbar (1990) and van Damme (1987). On the other hand, replicator dynamics need not even converge to an equilibrium or a cycle. For a discussion of chaotic dynamics in four strategy evolutionary games see Skyrms (1992, 1993).

In both evolutionary and economic game theory the independence assumptions of the classical theory are an unrealistic technical convenience. However, the introduction of correlation leads the two theories in different directions. In the game theory of von Neumann and Morgenstern and Nash, the choice of a mixed strategy is thought of as turning the choice of one's pure act over to some objective randomizing device. The player's choice is then just the choice of the probabilities of the randomizing device, for example, the choice of the bias of a coin to flip. The randomizing devices of different players are assumed to be statistically independent. The introduction of mixed strategies has the pleasant mathematical consequence of making a player's space of strategies convex and assuring the existence of equilibria in finite games. From a strategic point of view, the coin flip is important because it pegs the degrees of belief of other players who know the mixed act chosen. If each player knows the mixed acts chosen by other players, uses these probabilities together with independence to generate degrees of belief about what all the others will do, and if each player's mixed act maximizes (Savage) expected utility by these lights, then the players are at a Nash equilibrium.

This picture may seem unduly restrictive. Why could not some commonly known correlation exist between the individual players' randomizing devices? Players, in fact, might all benefit from using such a joint randomizing device. Or, to take a more radical line, if the only strategic

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[2]This is because mutation is not explicitly part of the replicator dynamics, and if the initial population is unmixed, no other strategies are around to replicate. The desirable step of incorporating mutation into the model leads from the simple deterministic dynamics discussed here to a stochastic process model. See Foster and Young (1990). The framework for correlation used in this paper can also be applied to stochastic replicator dynamics.

TABLE 6.1.

	Strategy 1	Strategy 2
Strategy 1	5, 1	0, 0
Strategy 2	4, 4	1, 5

importance of the randomizing devices is to peg other players' degrees of belief, why not dispense with the metaphor of flipping a coin and define equilibrium directly at the level of belief? From this perspective, the assumption of independence appears even more artificial. These lines of thought were introduced and explored in a seminal paper by Aumann (1974).

Aumann introduced the notion of a *correlated equilibrium*. Think of a joint randomizing device which sends each player a signal as to which pure acts to perform. This gives probabilities over each player's pure acts but these probabilities may be correlated. Such a device represents a *joint-correlated strategy*. Let us assume that all players know the joint probabilities generated by the device, but that when the signal goes out each player observes only her own signal and bases her degrees of belief about what the other players' pure acts will be on the probabilities conditional on this signal pegged by the joint randomizing device. If, under these assumptions, players have no regrets, that is, each player maximizes (Savage) expected utility, then the joint-correlated strategy is a correlated equilibrium. (Aumann 1987 shows how the notion can be subjectivized and viewed as a consequence of common knowledge of Bayesian rationality together with a common prior where Bayesian rationality is taken as ex post maximization of Savage expected utility.)

Notice that the definition of a correlated equilibrium involves a weak ratifiability concept. If players are at a correlated equilibrium, then each player's act will maximize expected utility for that player after the player is given the information that act was selected by the joint randomizing device. In this sense, players only play ratifiable strategies. However, this ratifiability concept crucially differs from the evolutionary one. In Aumann's correlated equilibrium, the relevant ratifiability concept is defined relative to Savage expected utility and in the context of correlated evolutionary game theory, the relevant ratifiability concept is defined relative to Jeffrey expected utility.

Two examples illustrate what can and cannot be a correlated equilibrium. Consider the two-person game in table 6.1 where row's payoffs are listed first and column's second. Only three uncorrelated Nash equilibria of the game exist: the pure equilibria where both players play strategy 1, and where both players play strategy 2, and a mixed equilibrium where each player plays each strategy with equal probability. Given the

TABLE 6.2.

	Moritz Cooperates	Moritz Defects
Max Cooperates	0.9, 0.9	0, 1
Max Defects	1, 0	0.6, 0.6

assumption of independence, each pair of strategies is played with prob-
ability 1/4, and each player has an expected payoff of 2.5. Both players
can do better than they do under this mixed strategy if they can play a
joint-correlated strategy. For example, they might flip a coin and both
play strategy 1 if heads comes up, otherwise both play strategy 2. This
is a correlated equilibrium which gives each player an expected payoff
of 3. In an even better correlated equilibrium, the joint-correlated strategy
chooses the strategy combinations $\langle 2,2 \rangle$, $\langle 1,1 \rangle$ and $\langle 2,1 \rangle$ with equal prob-
ability. Since each player is only informed of his own pure act, he has
no incentive to deviate. For instance, if row is informed that he does
strategy 2, he assigns equal probabilities to column doing strategies 1 and
2 and thus strategy 2 maximizes expected utility for him. In this corre-
lated equilibrium, each player gets an expected payoff of 3 1/3.

Correlated equilibrium does not help, however, with prisoner's di-
lemma (see table 6.2). Whatever the probability distribution of the joint-
correlated strategy, if Max is told to cooperate, cooperation will not max-
imize expected utility for him. This is a consequence of two facts: (1)
Defection strongly dominates cooperation. Despite whether Moritz co-
operates or defects, Max is better off to defect; and (2) the relevant ex-
pected utility is Savage expected utility rather than Jeffrey expected util-
ity. The only correlated equilibrium in prisoner's dilemma is the pure
strategy combination $\langle$ Defect, Defect $\rangle$. However, as we saw in example
1 of section 4, cooperation can be a strictly ratifiable and dynamically
strongly stable strategy in correlated evolutionary game theory providing
that the correlation of interactions is favorable enough. This example shows
how wide the gap is between the effects of correlation in evolutionary
game theory and in economic game theory. This is not to say that Aumann's
sort of correlated equilibrium may not also have a part to play in evo-
lutionary game theory, but only that the kind of correlation introduced
by nonrandom pairing is different.

7. Efficiency in Evolutionary Games. The example of the last section
generalizes. The prisoner's dilemma has captured the imaginations of phi-
losophers and political theorists because it is a simple prototype of a gen-
eral problem. Interacting individuals who attempt to maximize their own
payoffs may both end up worse off because of the nature of the inter-
action. Everyone would prefer to be a cooperator in a society of coop-

erators to a defector in a society of defectors. Universal cooperation makes everyone better off than universal defection, but cooperation is neither an evolutionarily stable strategy of the Maynard Smith evolutionary game nor a Nash equilibrium of the associated two-person noncooperative game.

Let us consider an arbitrary evolutionary game, given by a fitness matrix, and say that a strategy S_i is *strictly efficient* if in interaction with itself it has a higher fitness than any other strategy S_j in self-interaction: $U_{ii} > U_{jj}$. Thus if a strategy S_i is strictly efficient, a population composed of individuals all playing S_i will have greater average fitness than a population of individuals all playing another strategy S_j. One version of the general problem of social philosophy in this setting is that the adaptive process of evolution may prevent the fixation of strictly efficient strategies, and indeed drive them to extinction.

One route to efficiency in evolutionary games that has attracted wide interest involves the consideration of repeated games. Consider either an infinitely repeated series of games with discounted payoffs or equivalently an indefinitely repeated series of games with some constant probability of one more play as one moves along the series. In an evolutionary setting, each encounter between two individuals is assumed to consist of just such a series of repeated games. This approach has become widely known through the work of Axelrod and Hamilton on indefinitely repeated prisoner's dilemma. If the probability of one more play is high enough, Axelrod shows that the repeated game strategy of tit-for-tat, that is, initially cooperating and then doing what the other did the last time, is a Nash equilibrium. Fudenberg and Maskin (1986) have shown that efficient outcomes of one-shot games are sustainable as Nash equilibria of repeated games. Tit-for-tat is not, however, an evolutionarily stable strategy in the sense of Maynard Smith since the strategy "always cooperate" does as well against tit-for-tat as tit-for-tat does against itself and as well against itself as tit-for-tat does. The point generalizes to other repeated games. (See Farrell and Ware 1988. Also see Boyd and Loberbaum 1987.)

Two major difficulties, however, interfere with the repeated game approach to efficiency. One is that a wide variety of repeated game strategies—some inefficient—can be sustained in this way as equilibria in indefinitely repeated games. The second, more serious difficulty, is that the assumptions of the theorem never really apply. Individuals have some finite upper bound to their lifetimes and certainly a finite upper bound to the number of repetitions of a game with a given other individual. Under these conditions the relevant theorems fail. Tit-for-tat, for example, is no longer even a Nash equilibrium.

The discussion of this paper suggests another way to sustain effi-

TABLE 7.1.

	Strategy 1	Strategy 2	Strategy 3
Strategy 1	10	20	0
Strategy 2	20	10	0
Strategy 3	17	17	10

ciency—through correlation. Under the most favorable conditions of cor-
relation, gratifying results follow immediately:

> If there is a strictly efficient strategy and conditional pairing propor-
> tions are constant at $p(S_i|S_i) = 1$ for all i, then the strictly efficient
> strategy is strictly ratifiable and is globally stable in the replicator
> dynamics.[3]

Things are even slightly better than stated since one will not quite need
perfect correlation if the strategy in question is strictly efficient.

The situation is less simple and straightforward with respect to the ef-
ficiency of mixed or polymorphic populations. Clearly, correlation can
enhance efficiency here in interesting ways. Consider a system with the
fitnesses in table 7.1. If the interactions between population members are
uncorrelated, then a population consisting of equal proportions of strategy
1 and strategy 2 individuals has an average fitness of 15 and can be in-
vaded by strategy 3 individuals which have an average fitness of 17. Then
the uncorrelated replicator dynamics carry strategy 3 to fixation for an
average fitness of 10. However, if we allow for correlated encounters,
an anticorrelated population equally divided between strategy 1 individ-
uals and strategy 2 individuals with $p(S1|S2) = p(S2|S1) = 1$ is possible.
This population has a fitness of 20, and cannot be invaded by strategy 3
individuals no matter what their pairing proportions are. A small pertur-
bation of the population in the direction of strategy 2 (0.5 − ϵ strategy
1, 0.5 + ϵ strategy 2) does not allow enough strategy 1 players to main-
tain perfect anticorrelation. Assuming any anticorrelation consistent with
the population proportions, all of the strategy 1 players will interact with
strategy 2 players, but a few of the strategy 2 players will have to interact
with each other. This lowers the expected fitness of strategy 2 below that

[3]If S is strictly efficient and the conditional pairing proportions give perfect self-correlation,
then $U(S)$ and $U(S')$ are constant with $U(S) > U(S')$ for any S' different from S throughout
the space. Then, by definition, $U(S) > U$ everywhere except at the point of fixation of S
and S is strictly ratifiable.

Considering the replicator dynamics, since both $[U(S) − U]$ and $p(S)$ are positive
throughout the interior of the space, the replicator dynamics makes $dp(S)/dt$ positive
throughout the interior; $p(S)$ is a global Liapounov function. It assumes its unique maxi-
mum at the point of fixation of S and it is increasing along all orbits. It follows that the
point of fixation of S is a globally stable attractor in the replicator dynamics. See Boyce
and DiPrima (1977) or Hirsch and Smale (1974).

TABLE 7.2.

	Strategy 1	Strategy 2	Strategy 3
Strategy 1	10	20	0
Strategy 2	30	10	0
Strategy 3	17	17	10

of strategy 1. In like manner, an excess of strategy 1 players lowers the expected fitness of strategy 1 below that of strategy 2. Thus, under the assumption that anticorrelation is maintained consistent with the population proportions, this efficient polymorphic population is strongly dynamically stable in the correlated replicator dynamics.

Efficiency in polymorphic populations is, however, not always so straightforward. An efficient polymorphic population may fail to be in equilibrium in the correlated replicator dynamics, even assuming the most favorable correlation consistent with population proportions. Table 7.2 modifies the foregoing example by enhancing the fitness of $S2$ played against $S1$. Now at a population equally divided between $S1$ and $S2$ with perfect anticorrelated interactions, the fitness of $S2$ is 30, that of $S1$ is 20, and the average fitness of the population is 25. But since the fitness of $S2$ is higher than that of $S1$, the correlated replicator dynamics causes the proportion of $S2$ individuals to increase. This means that there are not enough $S1$ individuals to pair with all $S2$s, so some $S2$s must pair with each other, and the expected fitness of $S2$ goes down, as before. These effects come into equilibrium in a population of $1/3$ $S1$ and $2/3$ $S2$. This polymorphic population is strongly stable in the correlated replicator dynamics, but its average fitness is only 20 whereas at the $\langle 1/2, 1/2 \rangle$ polymorphism the average fitness of the population is 25. Moreover, $\langle 1/2, 1/2 \rangle$ polymorphic state Pareto dominates the $\langle 1/3, 2/3 \rangle$ state in the sense that $S2$ individuals have higher fitness in the former, while $S1$ individuals have equal fitness in both.

In summary, correlation completely transforms the question of efficiency in evolutionary game theory. With perfect self-correlation the replicator dynamics inexorably drives a strictly efficient strategy to fixation—even if that strategy is strongly dominated. With other types of correlation, efficient polymorphisms are possible which are not possible without correlation. However, the mere fact that correlation must be consistent with population proportions already circumscribes the situations in which the most favorable correlation can support efficient mixed populations. In more realistic cases, correlation will fall short of extreme values. (Why this is so raises the important question of the evolution of correlation mechanisms.) Nevertheless, the novel phenomena which stand out starkly in the extreme examples may also be found in more realistic ones.

8. Related Literature. A rich biological literature, largely initiated by the important work of Hamilton (1963, 1964, 1971) but going back at least to Wright (1921), deals with nonrandom interactions. Hamilton (1964) discusses both detection and location as factors which lead to correlated interactions. He already notes here that positive correlation is favorable to the evolution of altruism (see also Hamilton 1963). This point is restated in Axelrod (1981,1984) and Axelrod and Hamilton (1981), where a scenario with high probability of interaction with relatives is advanced as a possible way for tit-for-tat to gain a foothold in a population of "always defect". Fagen (1980) makes the point in a one-shot rather than a repeated game context. Hamilton (1971) develops models of assortative pairing (and dissortative pairing) in analogy to Wright's assortative mating. Eshel and Cavalli-Sforza (1982) further develop this theme with explicit calculation of expected fitnesses using conditional pairing probabilities. Michod and Sanderson (1985) and Sober (1992) point out that repeated game strategies in uncorrelated evolutionary game theory may be thought of as correlating devices with respect to the strategies in the constituent one-shot games. Extensive form games other than conventional repeated games could also play the role of correlating devices. Feldman and Thomas (1987) and Kitcher (1993) discuss various kinds of modified repeated games where the choice whether to play again with the same partner—or more generally the probability of another repetition—depends on the last play. The basic idea is already in Hamilton (1971), "Rather than continue in the jangling partnership, the disillusioned cooperator can part quietly from the selfish companion at the first clear sign of unfairness and try his luck in another union. The result would be some degree of assortative pairing" (p. 65). Gautier (1986) and Hirshleifer and Martinez Coll (1988) discuss perfect detection models. Robson (1990) considers selection of an efficient evolutionarily stable strategy in a repeated game context by introduction of a mutant who can send costless signals. This is done within the context of uncorrelated evolutionary game theory, with the signals inducing correlation in plays of the initial game embedded in the signaling game. The evolutionary selection of efficient equilibria in repeated games is also treated in Fudenberg and Maskin (1990) and Binmore and Samuelson (1992). Wilson (1980) discusses models where individuals interact within isolated subpopulations. Even if the subpopulations were generated by random sampling from the population as a whole and individuals pair at random within their subpopulations, the subpopulation structure can create correlation. The basic idea is already in Wright (1945, 417). Pollock (1989) explores consequences of correlation generated by Hamilton's population viscosity for the evolution of reciprocity where players are located on a spatial lattice. Myerson et al. (1991) develop a solution concept for evolutionary

games based on taking a limit as Hamilton's population viscosity goes to zero. Nowak and May (1992,1993) and Grim (1993) explore the effects of space in cellular automata models.

9. Conclusion. Correlated interactions are the norm in many biological situations. These may be a consequence of a tendency to interact with relatives (Hamilton's kin selection), of identification and discrimination, of spatial location, or of strategies established in repeated game situations (the reciprocal altruism of Trivers 1971 and Axelrod and Hamilton 1981). The crucial step in modifying evolutionary game theory to take account of correlations is merely to calculate expected fitness according to *The Logic of Decision* rather than *The Foundations of Statistics* (Savage 1954).

This means that strategies such as cooperation in one-shot prisoner's dilemma with a clone are converted to legitimate possibilities in correlated evolutionary game theory. It is not generally true that evolutionary-adaptive processes will lead the population to behave in accordance with the principles of economic game theory. The consonance of evolutionary and economic game theory only holds in the case of independence. When correlation enters, the two theories part ways. Correlated evolution can even lead to fixation of a strongly dominated strategy.

Positive correlation of strategies with themselves is favorable to the development of cooperation and efficiency. In the limiting model of perfect autocorrelation, evolutionary dynamics enforces a Darwinian version of Kant's categorical imperative, "Act only so that if others act likewise fitness is maximized". Strategies which violate this imperative are driven to extinction. If a unique (strictly efficient) strategy obeys it, then that strategy becomes fixed. In the real world, correlation is never perfect, but positive correlation is not uncommon. The categorical imperative is weakened to a tendency for the evolution of strategies which violate principles of individual rational choice in pursuit of the common good.

Correlation of interactions should continue to play a part, perhaps an even more important part, in the theory of cultural evolution (see Boyd and Richerson 1985, Cavalli-Sforza and Feldman 1981, and Lumsden and Wilson 1981). If so, then the special characteristics of correlation in evolutionary game theory will be important for understanding the evolution of social institutions. Contexts which involve both social institutions and strategic rational choice may call for the interaction of correlated evolutionary game theory with correlated economic game theory.

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